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CONSTRUCTIVE GEOMETRY

A SERIES OF MATHEMATICAL TEXTS

EDITED BY

EARLE RAYMOND HEDRICK

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CONSTRUCTIVE GEOMETRY

EXERCISES IN ELEMENTARY GEOMETRIC DRAWING

PREPARED UNDER THE DIRECTION

OF

EARLE RAYMOND HEDRICK

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PREFACE

THE saying is trite that students who enter formal courses in Euclidean Geometry have to learn both the strange methods of formal logic and the equally strange geometric forms.

A course to acquaint students with the elementary forms and constructions is valuable particularly to those who never go on to a more formal course, and it furnishes a basis for a truer comprehension by those who do go on.

Such courses are deservedly popular in Europe, but no good American geometric notebook exists. This is modeled after those long used successfully in England, some of which have been extensively used in America.

INSTRUMENTS

The student should have the following instruments:

1. A ruler, of which one edge is divided into inches and eighths of an inch.

Obtain if possible a ruler on which the metric units, centimeters and millimeters, are also marked.

2. A good pair of compasses, with pen and pencil points.

3. A semicircular protractor (see p. 46).

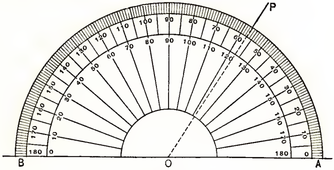

Fig. 1.

4. A drawing triangle, preferably one having angles of 90°, 60°, 30°.

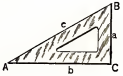

Fig. 2.

5. One soft and one medium hard pencil.

Any reasonably good case of drawing instruments will contain these and other desirable instruments.

In working out the problems of Section 12, pages 55–56, a supply of paper ruled in squares will be needed.

CONSTRUCTIVE GEOMETRY

CONSTRUCTIVE GEOMETRY

1. DEFINITIONS AND STATEMENTS OF FACT

A *point* is represented in a drawing by a dot or by some other small mark. We try to make the dot as small as we can, but it must be large enough to be seen. Mentally, we think of a point as having no width nor breadth, but it would be unreasonable to expect to make an actual dot without thickness or breadth.

Lines are drawn by moving the pencil point on the paper. As before, we think of a line mentally as without width, but the pencil marks we make in a drawing must be heavy enough to be seen.

Lines may be *straight* or *curved*. A good idea of a straight line is formed by means of a tightly stretched string, or by sighting between two points. A line may be tested for *straightness* by trying to fit the edge of a ruler to it. If the line is *curved*, the ruler will not fit it, but can be made to cross it *at least twice*.

A ruler may be tested for straightness by sighting along its edge. Another test for straightness of the ruler is to draw a line along its edge on paper and then turn the ruler over and fit the edge to the line in several positions. If the edge fits the line in all positions, the line is straight and the ruler is good.

A very good straight edge may be made by folding a piece of paper in the ordinary manner. Thus the edge of an envelope is usually quite straight.

2. MEASUREMENT OF DISTANCES

The scale on a ruler marked in inches is usually subdivided into eighths or into sixteenths of an inch. Twelve inches make a foot. Three feet make a yard. What other units of length in this English system do you know?

The scale on a ruler marked in centimeters is usually subdivided into tenths of a centimeter; that is, into millimeters. Ten millimeters make a centimeter. One hundred centimeters make a meter. A meter is about forty inches (more exactly, 39.37 inches).

The meter is the basis of the so-called *metric system.* A table of units in that system is usually given in arithmetic, and can be found also in any encyclopedia.

Familiarity with these units of length is gained by estimating the lengths of lines drawn on paper, and the lengths of actual objects, and by comparing these estimates with actual measurements with a ruler.

EXERCISES I

1. Estimate the length of each of the following lines, then measure each of them; note your error.

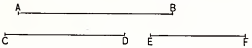

Fig. 3

Enter your results on page 3 in a table as follows:

	ESTIMATED LENGTH	MEASURED LENGTH	ERROR IN ESTIMATE
Line *AB*			
Line *CD*			
Line *EF*			

In making such a table, draw the lines neatly with a ruler. The corners can be made quite true by using the square corner of your drawing triangle.

One purpose in this course is to learn to draw such lines as those in this table neatly and accurately. To this end, use your compasses as well as your ruler, to make columns of equal width.

2. Measure the lengths of AB, BC, and AC in the following figure, separately. Add the measured lengths of AB and of BC, and see how nearly the sum comes to AC. Enter your results in a neatly drawn table.

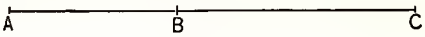

FIG. 4

3. Estimate the length of this page; its width; the length of the cover; the thickness of the book. Measure these same distances, and note the errors in your estimates. Enter all these numbers in a neatly drawn table.

4. Draw a straight line, and mark on it two points A and B which are $1\frac{1}{2}$ inches apart. Then mark a third point C so that $BC = 1\frac{1}{2}$ inches. Measure AC. Compare the length of $AB +$ the length of BC with the length of AC.

5. Draw a straight line and mark on it four points A, B, C, D, in that order, and so that $AB = 1\frac{1}{8}$ in., $BC = \frac{3}{4}$ in., $CD = 1\frac{1}{2}$ in. Measure AD and compare it with $AB + BC + CD$.

6. Draw a straight line, and mark on it five points A, B, C, D, E, one inch apart. Measure the lengths AB, AC, AD, AE, on your centimeter scale. Enter the results in a table similar to the following one.

	INCHES	CENTIMETERS	CENTIMETERS IN 1 INCH
AB	1		
AC	2		
AD	3		
AE	4		

The last column is to be filled out by dividing the number of centimeters in each of the lengths measured by the number of inches in it. Do your results agree precisely?

7. Draw a straight line and mark several points on it one centimeter apart. Measure the distances from the first point to each of the others in inches. Make a table similar to that of Ex. 6, with the headings in the order: *centimeters, inches, inches in 1 centimeter.*

8. Measure the width and length of your desk in feet and inches; reduce the results to inches. Measure the same distances in centimeters. Make a table similar to that of Ex. 6 for these measurements.

3. DIVISION OF A LENGTH INTO EQUAL PARTS

It is often convenient, for example in making such tables as that of Ex. 1, p. 2, to divide a length into two or more equal parts.

This can be done in several ways:

(*a*) *Arithmetically,* by measuring the given length, dividing the result into the desired number of parts, and then marking points at distances from each other equal to the quotient.

(*b*) *Mechanically,* by paper folding, or by some similar scheme. A sheet of paper may be folded very easily into two, four, eight, etc., equal strips.

(*c*) *Geometrically,* without first measuring the line. Later we shall see how to do this directly. Just now it can be done by trial by means of compasses. A very few trials will give a good result.

EXERCISES II

1. Draw a straight line and mark two points on it. Divide the length between the two points into two equal parts by each of the methods just mentioned. Which method seems most accurate?

2. Mark two points on a straight line as above. Divide that part of the line between the two points into three equal parts. Which of the methods described above can be used conveniently?

3. Divide the part of a line between two points on it into four equal parts. Which methods are convenient?

4. Draw a triangle of any shape. Measure each of its sides.

Divide one of the sides into two equal parts. Draw a line from the opposite corner of the triangle to the middle point of the side. Such a line in a triangle is called a *median.* Measure the length of this line.

5. Draw a triangle of any shape. Divide each of the sides into two equal parts. Draw all the possible medians.

How many medians are there? A test for the correctness of the drawing is that all the medians should pass through a common point.

6. Draw a triangle of any shape and divide each of its sides into two equal parts. Join the middle points of two of the sides by a straight line. Measure the length of this line. How does its length compare with the length of the third side of the triangle?

7. Draw, in Ex. 6, the other two lines which connect the middle point of one side with the middle point of another. Shade the interior of the small triangle formed by such lines.

Can you convince yourself that the original triangle is now divided into four small triangles, and that the sides of each of them are exactly half the length of the sides of the original one?

4. TO DRAW CIRCLES

The compasses are useful for measuring distances. They may be used for laying off on a line a distance equal to that between two points on another line.

Circles are usually drawn by means of compasses.

The point at which the fixed point of the compasses is placed is called the *center* of the circle. The line traced by the other (moving) point of the compasses is called the *circumference* of the circle, or simply the *circle.*

The distance from the center to the circumference is called the *radius* of the circle.

EXERCISES III

1. Open the compasses so that the distance between the two points is 1 inch. Draw a circle, keeping this opening fixed.

2. (*a*) Draw a straight line. About some point on this line draw a circle. In how many points does the circle cut the straight line?

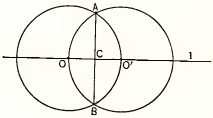

FIG. 5

(*b*) From one of the points where the circumference cuts the line draw a circle with the same radius as the original circle.

(*c*) Draw a line connecting the points where the two circles meet each other.

3. Draw two circles, each $2\frac{1}{2}$ in. in radius, about two points 4 in. apart on a line. Connect the two points in which these circles meet each other by a straight line.

4. Draw two equal circles, each 1 in. in radius about points 2 in. apart on a line. In how many points do these circles meet each other?

How far apart are the centers of two circles, if the circles just touch each other in one point?

5. (*a*) Draw any two circles which cut each other in two points, and draw the line joining their centers.

(*b*) Draw the line joining the two points where they cut each other.

These two lines are ***perpendicular*** to each other; that is, they come together at a square corner, which will fit the square corner of the drawing triangle.

6. (*a*) Draw two equal circles so that the circumference of one passes through the center of the other, and draw a straight line joining their centers.

(*b*) Join both centers to *one* of the points in which the circles cut each other.

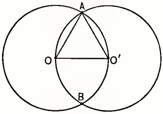

FIG. 6

The three lines form a triangle, all three of whose sides are equal. It is called an ***equilateral*** triangle.

7. In Fig. 5, $OC = CO'$ and $BC = AC$.

[This is true because the two circles are the same size. Hence we can pick up the whole figure, turn it over, and lay it down again with O' where O was and O where O' was. Likewise, the figure can be turned so that B falls where A was, and A where B was.]

Draw Fig. 5 again, and draw a circle about C as center with a radius CO. Does it pass through O'?

Draw a circle about C as center with a radius CA. Does it pass through B?

8. Draw an equilateral triangle (Ex. 6) and draw its medians (Ex. 4, p. 6). Notice that the median from A through the middle point of OO' should pass through B.

9. Redraw the figure for Ex. 6 (Fig. 6), but omit OO' and draw OB and BO'. The resulting four-sided figure $AOBO'$ has all four sides equal. It is called a ***rhombus***.

[The student should also try to see what figures are formed when the circles in Exs. 6, 7, 8, 9, are of unequal size, and when the center of one is not on the other.]

5. PERPENDICULARS

EXERCISES IV

To divide a line into two equal parts without measuring it, we may proceed almost as in Ex. 5, p. 12, by the following method:

1. (*a*) Draw a line and mark any two points on it; call them A and B.

(*b*) About A as center draw a circle which reaches nearly to B. About B as center draw a circle equal to the first one.

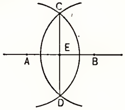

FIG. 7

(*c*) Connect the two points C and D in which these two circles cut each other by a new straight line. Mark the point E where this new straight line CD cuts the line AB.

This point E is halfway between A and B; that is, $AE = EB$.

The reasons for this are exactly similar to those given in Ex. 7, p. 12.

NOTE. After some practice, the student will see that it is not necessary to draw the full circles, but only portions of them, as in the printed figure.

2. Draw a figure which shows how to divide a line joining two points into *four* equal parts without measuring it.

3. The line CD of Ex. 1 is *perpendicular* to AB at E (see Ex. 5, p. 12).

To draw a line perpendicular to a given line, at a given point on that line, we may proceed as follows:

Draw a line AB and mark a point C on it. On opposite sides of C mark two

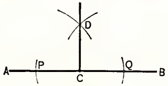

FIG. 8

points P and Q, so that $PC = CQ$. This can be done with the compasses.

Now follow the directions of Ex. 1 to get a new line CD; this new line is perpendicular to AB at C.

To draw a line perpendicular to a given line, through any point on the paper, we may proceed as follows:

4. (*a*) Draw a line AB and mark a point P not on the line.

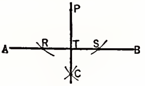

FIG. 9

(*b*) About P as center draw a circle which cuts the line in two points R and S.

(*c*) Now follow the directions of Ex. 1 to find a new line CT perpendicular to AB. This new line passes through P; it is the line desired.

5. Draw a straight line and mark two points A and B one inch apart on it. At A and at B draw lines perpendicular to AB.

Mark a point C on the perpendicular through A and one inch above A. Mark a point D on the perpendicular through B and one inch above B.

Connect C and D by a straight line. The figure $ABCD$ is a *square*.

6. Carry out the same directions as in Ex. 12, except that AC and BD are each one inch long, while AB is of different length. Such a figure is a *rectangle*.

In a *square*, each side is perpendicular to the sides next to it, and all the sides are of equal length.

In a *rectangle*, each side is perpendicular to the sides next to it, and each side is equal to the side opposite it.

In a *rhombus* (Ex. 9, p. 12) all four sides are of equal length, but the sides meet at any angle we may wish.

7. Draw a rectangle four inches long and $\frac{1}{2}$ inch high. Divide this rectangle into two equal rectangles by means of a perpendicular at the middle point of the base. Divide these rectangles again into two equal parts.

In this way very accurate blank forms, such as that used on p. 2, may be made.

8. A man goes 2 miles east, then 8 miles north, and finally 4 miles west. Draw a map of his route. Measure the distance from his starting point to his final position.

9. Draw a small map of the city block on which your school stands, and of each of the blocks next to it. Allow for widths of streets.

Measure the distance between two corners not on the same street.

6. PARALLELS

Two lines perpendicular to the same line will never meet each other. Such lines are called *parallels.*

EXERCISES V

1. Draw a line and mark several points $A, B, C, \cdots$ on it. Draw perpendiculars at each of the points $A, B, C, \cdots$ to the line. These new lines are all parallel.

2. *To draw a parallel to a given line through a given point:*

(*a*) Draw a line AB and mark a point P not on the line.

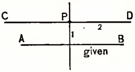

Fig. 10

(*b*) Draw a second line through P perpendicular to the given line and draw a third line through P perpendicular to the second line. (See Ex. 3, p. 15.)

The third line is parallel to the first, since both are perpendicular to the second line.

3. *To draw parallels with the drawing triangle.*

(*a*) Draw a line AB, and mark a point P, not on it.

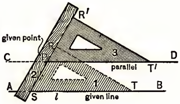

Fig. 11

(*b*) Lay the drawing triangle with any edge fitting the given line. Place a ruler (or a book) so as to fit either of the other edges of the drawing triangle.

(*c*) Hold the ruler (or the book) still, and slide the triangle along it, keeping the edge of the triangle tightly fitted against the ruler (or book) until the edge of the triangle which did fit against the given line comes near P.

(*d*) Draw a line through P along that side of the triangle which did fit the given line.

The new line is parallel to the given line.

4. Draw a picture of a picket fence by drawing two very long rectangles to represent the horizontal rails, and smaller rectangles to represent the slats.

5. Draw an ornamental border by drawing four rectangles one inside another about $\frac{1}{8}$ in. apart. This may be decorated by shading.

6. Draw nine parallels $\frac{1}{4}$ in. apart, and nine parallels perpendicular to them $\frac{1}{4}$ in. apart.

Shade the alternate squares to represent a checkerboard.

An ornamental border may be placed around the whole figure, as in Ex. 5.

7. Mark several points A, B, C, D, etc., on your paper. Draw lines through each of them parallel to the top edge of the paper by means of Ex. 3.

Likewise draw lines parallel to one side edge of the paper through A, B, C, D.

8. *To draw perpendiculars with the drawing triangle.*

The right-angled corner of the drawing triangle may be used directly, as in Ex. 1, p. 2. This gives blunt corners which are unsightly.

A better result is obtained by using the triangle as in the accompanying figure. The principle is almost the same as in Ex. 3, but the triangle and the ruler are placed

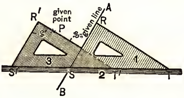

FIG. 12

as shown in figure: the number 1 marks the first position of the triangle, fitting against the given line AB; the number 2 marks the position of the ruler, fitting against the triangle; the number 3 marks the second position of the triangle, after it has slid along the ruler to the given point P. Draw such a figure.

9. Mark several points A, B, C, D, on your paper. Draw lines perpendicular to the top edge of your paper through each of these points by means of Ex. 8.

10. Mark a point P on your paper. Draw a line through P parallel to the top edge of the paper. Draw another line through P perpendicular to one of the side edges of your paper. If the paper is cut true, and if you have drawn accurately, these two lines should be exactly the same.

11. Draw a map showing at least four or five principal streets running east and west, and an equal number running north and south, in the city in which you live. Use Exs. 3 and 8. Measure the distance on this map between two important points not on the same street.

7. DRAWING ORNAMENTAL PATTERNS

Many ornamental designs may be made by means of the previous constructions. Some of these follow. Let the student try to devise others.

EXERCISES VI

1. Draw a rectangle 3 inches high and $1\frac{1}{2}$ inches wide. Draw a half circle whose center is the middle point of the top side of the rectangle and whose radius is $\frac{3}{4}$ inch.

This is the form of the so called Roman window, surmounted by a circular arch. Ornament it by lines drawn about $\frac{1}{8}$ inch from each side and by another half circle with the same center and with a radius about $\frac{1}{8}$ inch larger than that of the first circle.

Other lines may be drawn in an ornamental pattern to represent frames of glass.

2. (*a*) Draw an equilateral triangle as in Ex. 6, p. 12.

(*b*) About each corner of the base as center draw a portion of a circle joining the two remaining corners.

This is the basis of the so-called *Gothic* window. · Compare Ex. 13, p. 65.

3. Draw a square. About each of the corners as a center draw a circle whose radius is equal to one side of the square.

Various ornamental patterns may be formed by drawing only a part of each circle, and by shading the figures. Repeating the same design in several squares gives a striking effect.

4. Draw an equilateral triangle. Connect each corner by a straight line to the middle point of the opposite side. The three new lines thus drawn meet in one common point.

About this common point as center draw two circles, one of which passes through each of the corners, while the other just touches each side of the triangle.

5. Draw a triangle with unequal sides, and divide it into four small triangles as in Ex. 7, p. 6, by drawing the lines connecting the middle points of the sides.

Repeat the process, so that each of the four small triangles is divided into four still smaller.

By repeating this process and then shading the very smallest triangles alternately, a variety of interesting patterns may be formed.

6. (*a*) Mark three points P, Q, R one inch apart on a straight line. About each point as center draw a circle of radius one inch.

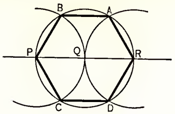

FIG. 13

(*b*) Mark the four points A, B, C, D, in which the middle circle cuts the others. Join the six points A, B, R, D, C, P by straight lines to form a six-sided figure.

This six-sided figure is called a ***regular hexagon.***

7. Redraw Fig. 13, and erase all except the central circle and the hexagon $ABPCDR$.

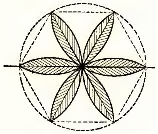

FIG. 14

About each of the six corners of the hexagon draw a circle which passes through the center Q of the original central circle, marking only the parts which lie inside the original circle. Shade parts of the figure to bring out the pattern vividly.

8. Draw each of the following figures:

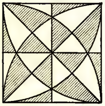

FIG. 15

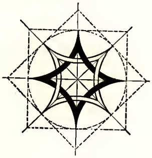

FIG. 16

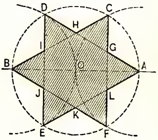

FIG. 17

8. MEASUREMENT OF ANGLES

The angle between two perpendicular lines is called a *right angle*. It is divided into 90 equal parts, each of which is called a degree (°). One sixtieth of a degree is called a *minute* (′); one sixtieth of a minute is called a *second* (″).

Since there are four complete right angles formed at the point where two perpendiculars meet, the total angle around the point is 4×90 degrees, or 360 degrees.

EXERCISES VII

1. Draw a square. How many right angles does it have? What is the sum of all of them put together?

2. Draw a square. Connect the opposite corners by straight lines.

These lines are called *diagonals*. The diagonals divide each of the angles at the corners into two equal parts. How large is each of these parts?

3. Draw an equilateral triangle. The size of each angle is 60°. What is the sum of all three of them?

4. Draw an equilateral triangle ABC, on the base AB. Extend the side AB beyond B. At B draw a perpendicular BD to AB, above AB.

How large is the angle ABC? How large is angle ABD? CBD?

5. *To move an angle from one position to another.*

(*a*) Draw an angle ABC, with its corner at B.

(*b*) Draw any new line MN and mark a point P on it.

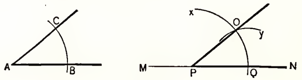

Fig. 18

(*c*) About A as center draw a circle of convenient size which cuts AB at B and AC at C.

(*d*) About P as center draw a circle of the same size as the preceding one, and mark a point Q where this circle cuts the line through P.

(*e*) With a radius equal to BC draw a circle about Q as center, and let O be a point where this circle cuts the one whose center is P.

The angle QPO is the same as the angle ABC moved into a new position.

5. To draw a clockface, first draw a circle and mark its center.

Then make successive angle of 30° (Ex. 4) whose corners are at the center of a circle, beginning with a vertical line through the center of the circle.

Mark the points along the circumference XII, I, II, III, IIII, V, VI, etc., as on a clockface.

This may be further ornamented as in the figure.

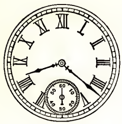

FIG. 19

6. Draw an angle of 45° (Ex. 2), and another angle of 30° (Ex. 4). Move the second angle to a new position so that its corner is at the corner of the 45° angle and so that one side of each lies in the same line.

What is the difference between the two angles? Shade the corresponding angle in your figure.

7. *To divide any angle into two equal parts.*

(*a*) Draw an angle ABC, with the corner at B.

(*b*) With B as center, and with any radius you please, draw a circle to cut AB at some point M and to cut CB at some point L.

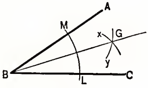

FIG. 20

(*c*) With L as center and with any radius you please, draw a circle.

(*d*) With M as center draw a circle of the same radius as that about L.

(*e*) Mark the point G where the last two circles cut each other.

(*f*) Draw the straight line BG.

Then BG divides $\angle ABC$ into two equal parts, so that $\angle CBG = \angle GBA$.

8. Draw a right angle. Divide it into two equal parts. How many degrees are there in each half?

9. Draw an angle of $22\frac{1}{2}$ degrees.

10. Draw an equilateral triangle (Ex. 6, p. 12). Divide one of its angles into two equal parts. How large is each half? (See Exs. 3, 4, p. 31.)

11. Draw an angle of 15°.

12. Draw any triangle.

Divide *each* of its angles into two equal parts.

The three dividing lines should pass through a single common point.

9. TRIANGLES

EXERCISES VIII

1. *To copy a triangle by means of its sides alone.*

(*a*) Draw any triangle ABC.

(*b*) Draw any line l. On l mark any point P, and lay off on l a distance PQ equal to AB with the compasses.

(*c*) About P as a center, draw a circle with a radius equal to AC; and about Q as center, draw a circle with radius equal to BC.

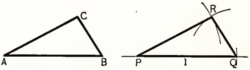

FIG. 21

(*d*) Mark a point R where the two circles meet, above l. Draw the lines PR, QR. Then the triangle PQR is exactly the same size and shape as ABC.

2. Draw a triangle whose sides are respectively 3 inches, 4 inches, 2 inches, long.

3. Choose any three trees in your school grounds, or in some park. Measure the distances between each pair, in feet. Draw a diagram on paper to represent their positions, using $\frac{1}{8}$ inch in the figure to every foot of actual distance.

By going from these three trees to a fourth one, and so on, a diagram may be made to show all the trees in a given yard. This process is called *triangulation*.

4. *To copy a triangle by means of one angle and two sides.*

(*a*) Draw a triangle ABC.

(*b*) Move the angle at B to any desired new position, by means of Ex. 5, p. 31.

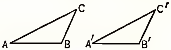

FIG. 22

(*c*) On the two sides of this new angle, lay off lengths equal to BA and BC, respectively; and join the ends of these lengths.

The new triangle formed is precisely the same size and shape as the triangle ABC.

5. Draw a triangle of which one side is 2 inches long, another side 3 inches long, and the angle between them is 60°. Measure the third side with your ruler. How long is it?

6. Draw a triangle with two sides 1 inch and 3 inches long, respectively, and with the angle between them equal to 90°. Measure the third side.

7. *To copy a triangle by means of one side and two angles.*

(*a*) Draw any triangle ABC.

(*b*) On any desired line lay off a length PQ equal to AB.

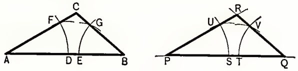

FIG. 23

(*c*) At P make an angle equal to $\angle BAC$, and at Q an angle equal to $\angle ABC$.

(*d*) Extend the sides of these new angles to form a triangle PQR.

This new triangle is precisely the same shape and size as ABC.

8. Draw a triangle of which one side is 2 inches long, with an angle of 45° at one end of that side and an angle of 60° at the other end.

Measure the two sides which were not given.

Measure the third angle of the triangle.

9. From a line 100 ft. long, near the shore of a lake, a surveyor measures the angles to an island in the water. If these angles are 90° and 30° respectively, how far is the island from the shore line?

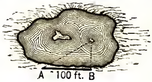

FIG. 24

Draw a figure with 1 inch in the figure equal to 25 feet of actual distance.

10. A flagpole FT stands on a level field. From a point S, which is 80 feet away from F, it is found that the angle FST is equal to 60°.

Draw a figure, making 1 inch in your figure equal to 20 feet of actual distance. Measure FT. How high is the flagpole?

11. From the window of a lighthouse known to be 75 feet above the water level, a boat is seen, at an angle of 15° below the horizontal line through the window. How far away is the boat?

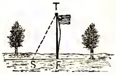

FIG. 25

12. A man walks 3 miles, turns 45° to his right, goes 2 miles, turns 90° to his right, and goes 1 mile. How far is he from his starting point?

13. How long a ladder is needed to reach the top of a wall 20 feet high, if the foot of the ladder must be 10 feet from the side of the wall?

14. Suppose a surveyor's notes of a triangular field read: Base AB = 100 feet, angle at A, 60°, angle at B, 90°. Draw a plan of the field, letting 1 inch in your plan equal 50 feet of actual distance, and measure the two other sides.

15. Suppose the notes of a triangular field to be $AB = 60$ yards, $AC = 45$ yards, angle at $A = 60°$. Draw a plan of the field, and find the length of BC.

16. A man walks 3 miles and turns 30° to his right. He then walks 4 miles, and turns 60° to his right, and again walks 3 miles. Find how far he is from his starting point.

17. A and B are two forts separated by a river. A man goes to a bridge, C, from one of the forts, and starts back to the other fort on a straight road making an angle of 30° with the road on the other side of the river. It is 6 miles from A to the bridge, and 8 miles from B to the bridge. How far apart are the forts?

18. A man 100 ft. from the base of a wireless station tower finds that the angle to the top of the tower is 60°. Draw a plan, and measure the height of the tower.

19. A baseball diamond is a square 90 ft. on each side, the bases being at the corners. (See Fig. 26.) If a ball is caught halfway between second base and third base, find the distance to first base; find the distance to home plate.

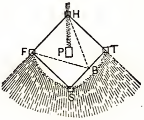

FIG. 26

20. An upright pole, 30 feet high, is stayed by a rope carried from the top to a point on the ground 20 feet from the foot of the pole. Make a diagram of this, using 1 inch = 10 ft., and find the length of the rope.

21. Directly east of where a man stands he can see a church tower which he knows to be two miles distant; due north he sees a standpipe which is $1\frac{3}{4}$ miles distant. Draw a plan, and find the distance from the church to the standpipe.

22. An automobile runs 25 miles north along a straight road, and then runs 17 miles due west. Draw a plan, and find how far the machine is from the starting point. How many miles would an aëroplane save, if it flew straight across?

23. In rowing across a river 78 yards wide, a man was carried downstream 23 yards. Represent this on a plan, and find the distance between the starting point and the landing point.

10. DIVISIONS OF A LINE. SIMILAR FIGURES

EXERCISES IX

1. *To divide a line into three equal parts.*

(*a*) Draw a line and mark two points A and B on it.

(*b*) From A draw any other line AC so that $\angle BAC$ is of any convenient size.

(*c*) On AC mark three points, P, Q, R, so that $AP = PQ = QR$.

(*d*) Draw the straight line BR.

(*e*) Draw parallels to BR through P and Q. (See Ex. 3, p. 21.)

These parallel lines divide AB into three equal parts.

Similarly a line AD may be divided into any desired number of equal parts by drawing a set of parallels from points $H, I, J, K, \cdots$, equally spaced on some other line.

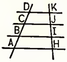

Fig. 27

2. Draw a line AB and divide it into five equal parts.

3. Draw a square. Divide this square into three equal rectangles, by lines parallel to its base, through points that divide one side into three equal parts.

4. (*a*) Draw a triangle of any shape.

(*b*) Divide each of its sides into three equal parts.

(*c*) Draw a new triangle, each of whose sides is equal to one third the corresponding side of the first triangle.

5. To reduce a figure in the ratio 1 : 3 means to make a new figure in which each line is one third the corresponding line in the given figure.

(*a*) Draw a rectangle and one diagonal of it.

(*b*) Reduce this figure in the ratio 1 : 3.

Figures are said to be **similar** to each other if one of them is the same as the other except that it is reduced or enlarged in size.

6. Divide a line 4 in. long into 5 equal parts; a line 7 in. long into 11 equal parts.

7. Draw a line 3 in. long, and divide it into 8 equal parts. Test afterwards by measurement.

8. Use the same method to bisect a line of any convenient length. Then bisect the line by the use of the compass, and see if the two results agree.

Drawings which represent large objects are always made on a reduced scale. The drawing is made *similar* to the object represented by reducing all dimensions in the same ratio. Often one inch in the drawing represents one foot on the object represented.

9. Make a drawing to represent a four-sided figure which has two sides parallel and one foot apart, the other two sides equal, but not parallel, and two feet long.

10. Draw a vertical cross section of a ditch which is 4 feet wide at the top, 2 feet wide at bottom, and 3 feet deep. Measure the length of the side.

11. Draw a square. On each side of this square draw an equilateral triangle. Join the vertices of these triangles, and show by measurement that the figure so formed is a square. What is the ratio of the side of this square to that of the original one?

12. A man measures a four-sided field. He finds that the diagonals bisect one another, and form an angle of 30° with each other. They are 60 yards and 80 yards, respectively. Find the length of each side of the field.

13. A ditch around a prison runs close up to the prison wall. A man finds that when he is 80 feet away from the outer edge of the ditch, the angle to the top of the prison wall is 45°, while at the edge of the ditch it subtends an angle of 60°. Find the width of the ditch.

14. House plans are usually drawn on such a scale that $\frac{1}{4}$ inch in the drawing represents one foot in the actual house. Such drawings are rather large, however.

Draw a map of the first floor of your home, showing all windows, doors, and partition walls, on a scale of $\frac{1}{8}$ inch to one foot.

15. Any given length may be multiplied by any given number geometrically.

Thus, given any definite length MN, let us multiply it by $3\frac{1}{2}$.

Draw any two lines AB and AC meeting at A at any convenient angle (say between 30° and 45°).

On AB mark points D and E so that $AD = 1$ inch and $DE = 3\frac{1}{2}$ inches.

On AC mark a point F so that $AF = MN$.

Connect D and F. Through E draw a line parallel to DF. This parallel meets AC at some point, say G.

Then $FG = 3\frac{1}{2} \times AF$ or $3\frac{1}{2} \times MN$.

16. Take a length $MN = 1\frac{3}{4}$ inches. Multiply it by $2\frac{1}{4}$ geometrically. Measure the resulting line. Multiply $1\frac{3}{4}$ by $2\frac{1}{4}$ by arithmetic. Is the answer the same as before?

11. THE PROTRACTOR

The protractor may be used to lay off any desired angle, as well as to measure angles.

EXERCISES X

1. From a point A, 30 feet from the base C, of a tree, CB, the angle CAB between the ground and the line from A to the top B of the tree is 31°. Find the height of the tree.

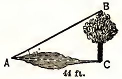

FIG. 28

The angle between a horizontal line, such as AC, and a line such as AB, from the observer A to a high object such as B is called the *angle of elevation* of B.

2. It is often difficult to reach the base of a tree or other object.

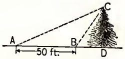

FIG. 29

From a point A the angle of elevation of the top of a pine is 25°. From a point B, which is 50 feet nearer to the tree, the angle of elevation is 55°.

Draw a figure by first drawing the line AB and then making the angles BAC and DBC by means of a protractor. Extend all lines to complete the figure, and measure DC.

3. Find the height of a statue, if the angles of elevation from two points, one of which is 20 feet nearer the statue than the other, are 35° and 45° respectively.

4. A roof is built with a pitch of one third; that is, the height above the plate to the ridge is one third the entire span.

Draw the figure accurately to scale. Measure the angle which the rafters make with the horizontal.

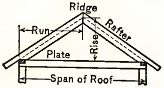

FIG. 30

5. The rafters of a roof make an angle of 35° with the horizontal, and the span is 32 feet. Draw a figure to scale and measure the rise. Find the pitch of the roof.

6. Draw two lines perpendicular to each other through the center of a circle. Mark the points where these lines meet the circumference of the circle, and join these points by straight lines.

The resulting figure is a square. It is said to be *inscribed* in the circle.

7. Divide the total angle (360°) about the center of a circle into five equal angles, by drawing lines which make angles equal to one fifth of 360° or 72°. Use the protractor.

Join these points by straight lines. The resulting five-sided figure is called a *regular pentagon*. It is inscribed in the circle.

8. A regular six-sided figure (hexagon) can be inscribed in a circle by drawing angles of 60° about the center of the circle.

Do this first by drawing angles of 60° as in Ex. 3, p. 31.

Draw the same figure, using your protractor.

9. Draw a regular eight-sided figure (octagon) inscribed in a circle.

10. Draw a regular nine-sided figure inscribed in a circle.

11. Draw a triangle of any shape and measure the three angles. Add your answers together. Is the sum 180°? If not, how much does it differ from 180°?

The true sum of the angles of any triangle is 180°.

12. Draw a triangle with a right angle at one corner, and an angle of 25° at another. Measure the third angle. Is the sum of all three angles 180°?

13. An angle less than a right angle is called an *acute* angle.

A triangle with one right angle is called a *right triangle.*

Draw a right triangle, and measure the two acute angles. What is their sum?

14. Draw a right triangle of which one acute angle is 36°. How large is the other acute angle? Measure it.

15. A vertical windmill tower 50 feet high stands on level ground. Find the angles of elevation of the top and middle point of the tower from a point on the ground 30 feet away from the base of the tower.

16. A flagstaff stands on top of a tower. At a distance of 80 feet from the base of the tower, the angle of elevation of the top of the tower is found to be 55°, while the angle of elevation of the top of the flagstaff is 75°. Find the length of the flagstaff and the height of the tower.

17. A shore battery has an effective range of 4 miles. A ship is fired upon while $2\frac{1}{2}$ miles NW of the battery; she then turns NE, and goes 2 miles. There she anchors for repairs, thinking herself out of range. Is she?

18. From a point P I walk east 2 miles, then turn SW and walk 3 miles. I then return directly to P. How far do I walk all together?

19. A balloon is held captive by a rope 300 yards long. It drifts in the wind until its angle of elevation, from the place where the rope is tied, is 65°. How high is the balloon above the ground?

20. A tower stands on a rock; a man 100 yards away from the foot of the rock finds the angle of elevation of the *foot* of the tower to be 25°. When he is 200 yards away, he finds the angle of elevation of the *top* of the tower to be 25°. Find the heights of the rock and of the tower.

21. The *angle of depression* of an object below the observer means the angle between a horizontal line and a line depressed downwards passing through the object.

At the top of a mountain it is found that the angle of depression of a neighboring peak is 5°. If the difference in the heights of the two mountains is known to be 500 feet, find the distance between the peaks.

22. A man wishing to find the distance of an enemy's fort measures a base of 100 yards, and finds that the angles at the ends of the base are each 70°. Find how far the fort is from either end of the base, and measure the third angle.

23. Let A and B be two inaccessible objects, and C a point from which they can both be seen. The angle DCE is 135°. I measure CD and CE, each 100 yards, and observe the angles CDA and ACD, and find them to be 30° and 80°. I measure the angles BCE and CEB, and find them to be each $67\frac{1}{2}°$. Find the distance between A and B.

24. If sand is poured out carefully in a heap, the angle which the side of the pile makes with the horizontal is always the same for the same grade of sand. This angle is called the *angle of repose.*

Measure the height and the width of a small pile of sand carefully.

Draw a figure to represent a vertical section of such a pile, and find the angle of repose.

25. Draw a figure to scale to represent a pile 12 feet wide of the sand used in Ex. 24. Measure from your figure the height of the pile. Then find its volume from the formula $\frac{1}{3}\pi r^2 h$, where $\pi = 3\frac{1}{7}$, r = radius of base, h = height.

12. SQUARED PAPER — AREAS

Squared paper is paper ruled into little squares. It may be bought already ruled.

Usually the smallest squares are made one tenth of an inch on each side. Larger squares whose sides are one inch long are usually marked by heavier lines.

EXERCISES XI

1. Copy the following designs by drawing more heavily some of the lines on a sheet of squared paper, and by drawing in the diagonals.

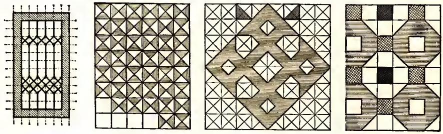

FIG. 31

2. Draw a triangle on squared paper, and estimate its area.

Remember that there are 100 small squares whose sides are $\frac{1}{10}$ inch, in one square inch. Hence each small square counts as $\frac{1}{100}$ square inch if the paper is ruled in tenths of an inch.

A good plan is to count all the squares which are wholly inside a figure, and then to add *half* the squares which are partly inside and partly outside.

3. Draw any rectangle on squared paper. Draw a similar rectangle in the ratio 1 : 2, and show by counting the squares that the area of the larger one is four times the area of the smaller one.

The area of a rectangle in square inches is equal to the number of inches in its length times the number of inches in its height.

4. Draw any circle. Draw a circle of twice the radius. Compare their areas.

5. Draw a square and inscribe a circle in it. Compare the area of the circle with that of the square.

The area of a circle is found to be about $3\frac{1}{7}$ times the square of the radius. (More accurately $3.1416 \times (\text{radius})^2$.)

The area of the square in which the circle is inscribed is evidently $4 \times$ the square of the radius. The two areas should therefore be in the ratio $3\frac{1}{7}$ to 4, nearly.

6. A good practical way to enlarge a figure is to draw it on squared paper and then redraw it, taking as many tenths of inches in place of one tenth inch as is desired for the enlargement.

Draw a figure of any kind, and enlarge it by this method in the ratio 1 : 5.

7. Make an outline map of the state of Michigan, twice the size of that in your atlas, by using squared paper. Verify the correctness of your copy by measuring distances between other points than those used to make your figure.

8. Make a map of the Mississippi and Ohio rivers from Quincy, Ill., and Cincinnati, to Memphis, half or twice the size of the map in your geography. Test the correctness of your drawing.

9. On a map whose scale is 5 miles to the inch, a piece of land is represented by an area of 24 square inches. What is the area of the land?

10. Squared paper is very useful for making plans of houses and other objects.

Draw a plan of the first floor of your home on squared paper, taking one small division ($\frac{1}{10}$ inch) to represent one foot in the actual house.

11. Squared paper may be used to draw maps by measuring the distances to important points from two side lines at right angles to each other.

Draw a map of your school grounds on squared paper, taking one small division ($\frac{1}{10}$ inch) to represent five or ten feet, as is convenient. Mark all trees and buildings by measuring the distance from each of them to the front sidewalk and to the side line of the lot.

12. We frequently wish to find the areas of very irregular objects that occur in nature. Thus the areas of leaves are important in agriculture, since the amount of growing power of a plant depends on the area of its leaves.

Press an irregular leaf on squared paper, and determine its area after tracing its edge in pencil.

13. Stick two pins firmly in a sheet of squared paper, about one inch apart. Around them tie loosely a loop of stout thread about three inches long. Stretch this loop taut with the point of your pencil, and move the pencil around.

The curve formed is called an *ellipse*. Find its area.

MISCELLANEOUS APPLICATIONS

1. Draw a half circle. Draw two smaller half circles whose diameters are the

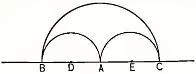

Fig. 32

two radii of the larger circle.

This figure is used as the basis of many ornamental designs.

2. Copy accurately each of the following designs enlarged in the ratio 1 : 4. These designs are all based on the construction of Ex. 1.

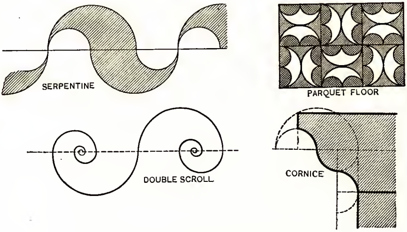

Fig. 33

3. Draw a regular octagon (Ex. 9, p. 51), and draw all the possible diagonals. How many are there?

4. Draw a polygon of sixteen equal sides inscribed in a circle. Draw all the possible diagonals.

A favorite test of technical skill in using drawing instruments is to draw on a large sheet of paper a polygon of sixty-four equal sides inscribed in a circle, and to draw all its diagonals. If this figure is attempted at all, a long time should be allowed for its completion, since there are 1952 diagonals.

In general, each corner of a polygon can be connected by a diagonal to all but three of the corners, — itself and the two nearest it.

5. To find the center of a given circle whose center is unknown.

(*a*) Draw a circle (or a portion of one), keeping the center unmarked by putting a small piece of pasteboard under the compass point.

(*b*) Mark any points A, B, C on the circle, and draw the lines AB and BC.

(*c*) Draw a line perpendicular to AB at its middle point. Also draw a line perpendicular to BC at its middle point.

(*d*) Extend these two perpendiculars to meet at a point O.

This point O is the center of the circle.

6. Given three points A, B, C in the plane, draw a circle through them. Do not put A, B, C in the same straight line.

7. Draw a triangle of any form, and draw a circle that passes through its three corners.

Such a circle is called a *circumscribed* circle.

8. To round off a sharp corner by a circle touching both sides of an angle.

(*a*) Draw any angle ABC.

(*b*) Divide the angle into two equal parts. (See Ex. 7, p. 32.)

(*c*) From any point P in the dividing line, draw a perpendicular PD to one of the sides of the angle BC, meeting that side at a point D.

(*d*) About P as center, with a radius equal to the perpendicular PD, draw a circle.

9. A street car line turns a corner at which two streets are perpendicular. To turn a circle of 20 feet radius is inserted. Draw a diagram of the track, if the width of the track is 4 feet.

Such designs for street car lines and railroads are sometimes very complex. Cases in which the angle between LN and NQ is not a right angle, and cases in which there are two or more turns, occur frequently. Additional figures of this kind may be made if there is time for it.

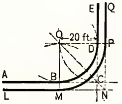

FIG. 34

10. Draw any triangle and divide each of the angles into two equal parts.

These three dividing lines meet in a point.

With this point as center, draw a circle that touches each side of the triangle.

11. The belt of a sewing machine runs over two wheels whose centers are 18 inches apart. The diameters of the wheels are 12 inches and 4 inches respectively.

Draw this figure to scale. The parts of the belt between the wheels can be drawn by placing the ruler so as to touch both circles.

Measure in degrees with the protractor the portion of the surface of each wheel in contact with the belt.

12. Draw the following patterns enlarged 1 : 4. Explain how each one is drawn.

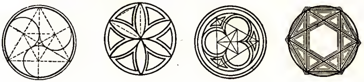

FIG. 35

13. Copy the following ornamental designs for Gothic windows.

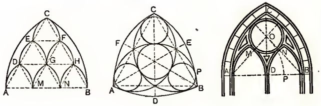

FIG. 36

14. Light is reflected from a mirror so that the reflected ray makes the same angle with the mirror that the original ray makes. Copy this figure.

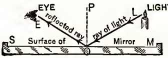

FIG. 37

15. Draw a figure to represent two mirrors that stand at an angle of 45° and show that a ray of light which strikes one of them parallel to the other is reflected exactly to its source.

16. Two mirrors stand at an angle of 60°. Draw a figure to show how a ray of light is reflected which strikes one of these mirrors parallel to the other one.

Figures may be drawn to illustrate the following principle: Any point of an object and its image in a mirror are equally distant from the mirror, and the line joining object and image is perpendicular to the mirror.

17. The following table shows the notes taken by a surveyor in surveying a broken line *ABCDEF*. Draw a map of this to scale.

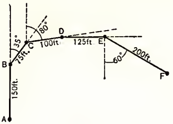

Fig. 38

Station	A (to B)	B (to C)	C (to D)	D (to E)	E (to F)
Direction	N	N 35° E	N 80° E	E	S 60° E
Distance	150 ft.	75 ft.	100 ft.	125 ft.	200 ft.

N 35° E means 35° to the east of true north.

S 60° E means 60° to the east of true south.

Measure the distance and the direction from A to each of the points C, D, E, F.

18. These are the notes for a railway switch, beginning at the point where the switch is to leave the main line. Lay out a map of the switch, using ruler and protractor, to the scale of 1000 feet to the inch.

Stations	1	2	3	4	5	6
Bearings	N 20° E	N 40° E	N 45° E	N 60° E	E	E 15° S
Distances	500 ft.	1250 ft.	1000 ft.	2000 ft.	500 ft.	1000 ft.

Surveying is often carried on in the manner described on this page. The surveyor ordinarily uses a telescope mounted on a tripod to measure the angles accurately. Reasonably good results can be found, however, without any special instruments, by the simple expedient of sighting across pins stuck in a level board, and arranging the pins so that the line of sight from the middle pin across either one of two others passes through a point located at one of the two corners of the route of the survey. A group of students can readily survey in this manner the bank of some stream or the track of some railroad, and then draw a map of it.

Ships are often navigated on short trips by a similar scheme, keeping the direction by means of the ship's compass, and sailing in straight lines as long as is convenient.

19. An egg-shaped drainage channel as shown in Fig. 39 is formed by four circular arcs. The circles ABC and DEF touch in G and the circular arcs AF and CD touch both circles, AC being a diameter of the larger circle. Make a copy of the figure to represent the case in which the radii of ABC and DEF are 2 feet and 1 foot respectively, choosing the centers of the circles AF and CD by trial, somewhere on AC extended.

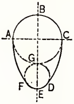

Fig. 39

[To locate the center of AF, for example, accurately, proceed as follows. Denote by M and N, respectively, the centers of the circles ABC and DEF. Connect N to a point P on AC with $AP = GN$. Draw NX so that $\angle PNX = \angle MPN$. The true center of AF is the intersection of AC and NX.]

20. Make a copy of the adjoining figure, which represents a siphon carrying water from one vessel to another.

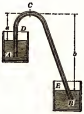

Fig. 40

21. Make a copy of the adjoining figure, which represents the outlines of a steam engine.

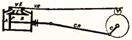

Fig. 41

A variety of geometric outline drawings of engines can be found in encyclopedias, books on engines, and even in advertisements. The student may discover such a drawing and copy it.

22. Make a copy of the adjoining figure, which represents the action of a magic lantern in throwing a picture on a screen.

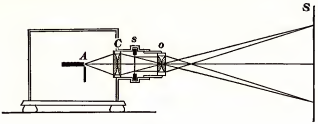

Fig. 42

Textbooks on physics, and those on geometrical optics, contain a great variety of figures of this sort. The action of lenses, cameras, telescopes, microscopes, the human eye, field glasses, etc., can be illustrated vividly by such figures.

23. Make a copy of the following figure, which represents the action of a force-pump.

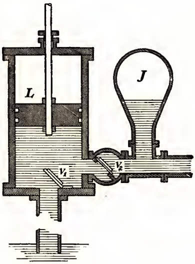

Fig. 43

The circular top of the equalizing tank J may be drawn accurately by means of Ex. 8, p. 62, so that there will be no break in the smoothness of the surface where it joins the straight sides. The purpose of this tank is to equalize the flow by means of the varying compression of the air in the tank.

24. Make a copy of the adjoining figure which represents the outlines of an electric dynamo.

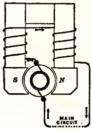

FIG. 44

25. Draw on a larger scale the following diagrams. If these are cut out of paper and folded along the dotted lines they can be closed into solid figures called the ***regular solids.***

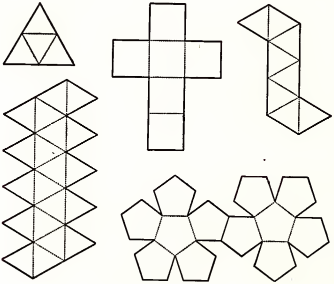

FIG. 45

In actually making such models, little flaps may be left attached to the edges which are to be joined together later. Such flaps are very convenient in pasting the figures together to form the solids.

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